17. Increasing and Decreasing Functions

Exercise 17.1

1. Question

Prove that the function $f(x) = \log_e x$ is increasing on $(0, \infty)$.

Answer

let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in $(0, \infty)$

2. Question

Prove that the function $f(x) = \log_a x$ is increasing on $(0, \infty)$ if $a > 1$ and decresing on $(0, \infty)$, if $0 < a < 1$.

Answer

case I

When $a > 1$

let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in $(0, \infty)$

case II

When $0 < a < 1$

$f(x) = \log_a x = \frac{\log x}{\log a}$

when $a < 1 \Rightarrow \log a < 0$

let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ $[\because \log a < 0]$

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

3. Question

Prove that $f(x) = ax + b$, where a, b are constants and $a > 0$ is an increasing function on R.

Answer

we have,

$f(x) = ax + b, a > 0$

let $x_1, x_2 \in$ R and $x_1 > x_2$

$\Rightarrow ax_1 > ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b > ax_2 + b$ for some b

$\Rightarrow f(x_1) > f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

So, f(x) is increasing function of R

4. Question

Prove that f(x) = ax + b, where a, b are constants and a < 0 is a decreasing function on R.

Answer

we have,

$f(x) = ax + b, a < 0$

let $x_1, x_2 \in$ R and $x_1 > x_2$

$\Rightarrow ax_1 < ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b < ax_2 + b$ for some b

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, f(x) is decreasing function of R

5. Question

Show that $f(x) = \frac{1}{x}$ is a decreasing function on (0, ∞).

Answer

we have

$f(x) = \frac{1}{x}$

let $x_1, x_2 \in (0, \infty)$ We have, $x_1 > x_2$

$\Rightarrow \frac{1}{x_1} < \frac{1}{x_2}$

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, f(x) is decreasing function

6. Question

Show that $f(x) = \frac{1}{1+x^2}$ decreases in the interval [0, ∞) and increases in the interval (-∞, 0].

Answer

We have,

$f(x) = \frac{1}{1+x^2}$

Case 1

When $x \in [0, \infty)$

Let $x_1, x_2 \in (0, \infty]$ and $x_1 > x_2$

$\Rightarrow x_1^2 > x_2^2$

$\Rightarrow 1 + x_1^2 > 1 + x_2^2$

$\Rightarrow \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow f(x_1) < f(x_2)$

$\therefore$ f(x) is decreasing on$[0,\infty)$.

Case 2

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

$\Rightarrow x_1^2 < x_2^2$

$\Rightarrow 1 + x_1^2 < 1 + x_2^2$

$\Rightarrow \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow f(x_1) > f(x_2)$

$\therefore$ f(x) is increasing on$(-\infty,0]$.

Thus, f(x) is neither increasing nor decreasing on R.

7. Question

Show that $f(x) = \frac{1}{1+x^2}$ is neither increasing nor decreasing on R.

Answer

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case 1

When $x \in [0, \infty)$

Let $x_1 > x_2$

$\Rightarrow x_1^2 > x_2^2$

$\Rightarrow 1 + x_1^2 > 1 + x_2^2$

$\Rightarrow \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow f(x_1) < f(x_2)$

$\Rightarrow \therefore$ f(x) is decreasing on$[0,\infty)$.

Case 2

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

⇒ $x_1^2 < x_2^2$

⇒ $1 + x_1^2 < 1 + x_2^2$

⇒ $\frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

⇒ $f(x_1) > f(x_2)$

∴ f(x) is increasing on(-∞,0].

Thus, f(x) is neither increasing nor decreasing on R.

8. Question

Without using the derivative, show that the function f(x) = | x | is

A. strictly increasing in (0, ∞)

B. strictly decreasing in (-∞, 0).

Answer

We have,

f(x) = |x| = $\{x, x > 0$

(a)Let $x_1, x_2 \in (0, \infty)$ and $x_1 > x_2$

⇒ $f(x_1) > f(x_2)$

So, f(x) is increasing in (0, ∞)

(b) Let $x_1, x_2 \in$ (-∞, 0)and $x_1 > x_2$

⇒ $-x_1 < -x_2$

⇒ $f(x_1) > f(x_2)$

∴ f(x) is strictly decreasing on(-∞, 0).

9. Question

Without using the derivative show that the function f(x) = 7x - 3 is strictly increasing function on R.

Answer

Given,

f(x) = 7x – 3

Lets $x_1, x_2 \in$ R and $x_1 > x_2$

⇒ $7x_1 > 7x_2$

⇒ $7x_1 - 3 > 7x_2 - 3$

⇒ $f(x_1) > f(x_2)$

∴ f(x) is strictly increasing on R.

Exercise 17.2

1 A. Question

Find the intervals in which the following functions are increasing or decreasing.

f(x) = 10 – 6x – $2x^2$

Answer

Given:- Function $f(x) = 10 - 6x - 2x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = 10 - 6x - 2x^2$

$\Rightarrow f'(x) = \frac{d}{dx}(10 - 6x - 2x^2)$

$\Rightarrow f'(x) = -6 - 4x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -6 - 4x > 0$

$\Rightarrow -4x > 6$

$\Rightarrow x < -\frac{6}{4}$

$\Rightarrow x < -\frac{3}{2}$

$\Rightarrow x \in (-\infty, -\frac{3}{2})$

Thus f(x) is increasing on the interval $\left(-\infty, -\frac{3}{2}\right)$

Again, For f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow -6 - 4x < 0$

$\Rightarrow -4x < 6$

$\Rightarrow x > -\frac{6}{4}$

$\Rightarrow x > -\frac{3}{2}$

$\Rightarrow x \in (-\frac{3}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{3}{2}, \infty)$

1 B. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^2 + 2x - 5$

Answer

Given:- Function $f(x) = x^2 + 2x - 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = x^2 + 2x - 5$

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 5)$

$\Rightarrow f'(x) = 2x + 2$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 2x + 2 > 0$

$\Rightarrow 2x < -2$

$\Rightarrow x < -\frac{2}{2}$

$\Rightarrow x < -1$

$\Rightarrow x \in (-\infty, -1)$

Thus f(x) is increasing on interval $(-\infty, -1)$

Again, For f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow 2x + 2 < 0$

$\Rightarrow 2x > -2$

$\Rightarrow x > -\frac{2}{2}$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

Thus f(x) is decreasing on interval $x \in (-1, \infty)$

1 C. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 6 - 9x - x^2$

Answer

Given:- Function $f(x) = 6 - 9x - x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 6 - 9x - x^2$

$\Rightarrow f'(x) = \frac{d}{dx}(6 - 9x - x^2)$

$\Rightarrow f'(x) = -9 - 2x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -9 - 2x > 0$

$\Rightarrow -2x > 9$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x \in (-\infty, -\frac{9}{2})$

Thus f(x) is increasing on interval $\left(-\infty, -\frac{9}{2}\right)$

Again, For f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow -9 - 2x < 0$

$\Rightarrow -2x < 9$

$\Rightarrow x > -\frac{9}{2}$

$\Rightarrow x > -\frac{9}{2}$

$\Rightarrow x \in (-\frac{9}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{9}{2}, \infty)$

1 D. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 - 12x^2 + 18x + 15$

Answer

Given:- Function $f(x) = 2x^3 - 12x^2 + 18x + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 - 12x^2 + 18x + 15$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 12x^2 + 18x + 15)$

$\Rightarrow f'(x) = 6x^2 - 24x + 18$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24x + 18 = 0$

$\Rightarrow 6(x^2 - 4x + 3) = 0$

$\Rightarrow 6(x^2 - 3x - x + 3) = 0$

$\Rightarrow 6(x - 3)(x - 1) = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 3 , 1$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

and $f'(x) < 0$ if $1< x < 3$

Thus, f(x) increases on $(-\infty,1) \cup (3, \infty)$

and f(x) is decreasing on interval $x \in (1,3)$

1 E. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 5 + 36x + 3x^2 - 2x^3$

Answer

Given:- Function $f(x) = 5 + 36x + 3x^2 - 2x^3$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 5 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = \frac{d}{dx}(5 + 36x + 3x^2 - 2x^3)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3 , - 2$

clearly, $f'(x) > 0$ if $-2< x < 3$

and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2,3)$

and f(x) is decreasing on interval $(-\infty,-2) \cup (3, \infty)$

1 F. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 8 + 36x + 3x^2 - 2x^3$

Answer

Given:- Function $f(x) = 8 + 36x + 3x^2 - 2x^3$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 8 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = \frac{d}{dx}(8 + 36x + 3x^2 - 2x^3)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3 , - 2$

clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2,3)$

and f(x) is decreasing on interval $(-\infty,-2) \cup (3, \infty)$

1 G. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 5x^3 - 15x^2 - 120x + 3$

Answer

Given:– Function $f(x) = 5x^3 - 15x^2 - 120x + 3$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 5x^3 - 15x^2 - 120x + 3$

$\Rightarrow f'(x) = \frac{d}{dx}(5x^3 - 15x^2 - 120x + 3)$

$\Rightarrow f'(x) = 15x^2 - 30x - 120$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 15x^2 - 30x - 120 = 0$

$\Rightarrow 15(x^2 - 2x - 8) = 0$

$\Rightarrow 15(x^2 - 4x + 2x - 8) = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = 4 , - 2$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$

and f’(x) < 0 if –2< x < 4

Thus, f(x) increases on (–∞,–2) ∪ (4, ∞)

and f(x) is decreasing on interval x ∈ (–2,4)

1 H. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^3 - 6x^2 - 36x + 2$

Answer

Given:- Function $f(x) = x^3 - 6x^2 - 36x + 2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f’(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f’(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f’(x)

(iii) Put f’(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^3 - 6x^2 - 36x + 2$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 - 36x + 2)$

$\Rightarrow f'(x) = 3x^2 - 12x - 36$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 12x - 36 = 0$

$\Rightarrow 3(x^2 - 4x - 12) = 0$

$\Rightarrow 3(x^2 - 6x + 2x - 12) = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow (x - 6)(x + 2) = 0$

$\Rightarrow x = 6, - 2$

clearly, f’(x) > 0 if x < –2 and x > 6

and f’(x) < 0 if –2< x < 6

Thus, f(x) increases on (–∞,–2) ∪ (6, ∞)

and f(x) is decreasing on interval x ∈ (–2,6)

1 I. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 - 15x^2 + 36x + 1$

Answer

Given:- Function $f(x) = 2x^3 - 15x^2 + 36x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$

$\Rightarrow f'(x) = 6x^2 - 30x + 36$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 30x + 36 = 0$

$\Rightarrow 6(x^2 - 5x + 6) = 0$

$\Rightarrow 3(x^2 - 3x - 2x + 6) = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

$\Rightarrow x = 3, 2$

clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$

and $f'(x) < 0$ if $2 < x < 3$

Thus, f(x) increases on $(-\infty, 2) \cup (3, \infty)$

and f(x) is decreasing on interval $x \in (2,3)$

1 J. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 + 9x^2 + 12x + 20$

Answer

Given:- Function $f(x) = 2x^3 + 9x^2 + 12x + 20$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 + 9x^2 + 12x + 20$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 + 9x^2 + 12x + 20)$

$\Rightarrow f'(x) = 6x^2 + 18x + 12$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 + 18x + 12 = 0$

$\Rightarrow 6(x^2 + 3x + 2) = 0$

$\Rightarrow 6(x^2 + 2x + x + 2) = 0$

$\Rightarrow x^2 + 2x + x + 2 = 0$

$\Rightarrow (x + 2)(x + 1) = 0$

$\Rightarrow x = -1, -2$

clearly, $f'(x) > 0$ if $-2 < x < -1$

and $f'(x) < 0$ if $x < -1$ and $x > -2$

Thus, f(x) increases on $x \in (-2,-1)$

and f(x) is decreasing on interval $(-\infty, -2) \cup (-2, \infty)$

1 K. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 - 9x^2 + 12x - 5$

Answer

Given:- Function $f(x) = 2x^3 - 9x^2 + 12x - 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 - 9x^2 + 12x - 5$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 9x^2 + 12x - 5)$

$\Rightarrow f'(x) = 6x^2 - 18x + 12$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 18x + 12 = 0$

$\Rightarrow 6(x^2 - 3x + 2) = 0$

$\Rightarrow 6(x^2 - 2x - x + 2) = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 1, 2$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 2$

and $f'(x) < 0$ if $1 < x < 2$

Thus, f(x) increases on $(-\infty, 1) \cup (2, \infty)$

and f(x) is decreasing on interval $x \in (1,2)$

1 L. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 6 + 12x + 3x^2 - 2x^3$

Answer

Given:- Function $f(x) = -2x^3 + 3x^2 + 12x + 6$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = -2x^3 + 3x^2 + 12x + 6$

$\Rightarrow f'(x) = \frac{d}{dx}(-2x^3 + 3x^2 + 12x + 6)$

$\Rightarrow f'(x) = -6x^2 + 6x + 12$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow -6x^2 + 6x + 12 = 0$

$\Rightarrow 6(-x^2 + x + 2) = 0$

$\Rightarrow 6(-x^2 + 2x - x + 2) = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

clearly, $f'(x) > 0$ if $-1 < x < 2$

and $f'(x) < 0$ if $x < -1$ and $x > 2$

Thus, f(x) increases on $x \in (-1, 2)$

and f(x) is decreasing on interval $(-\infty, -1) \cup (2, \infty)$

1 M. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 - 24x + 107$

Answer

Given:- Function $f(x) = 2x^3 - 24x + 107$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 - 24x + 107$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 107)$

$\Rightarrow f'(x) = 6x^2 - 24$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24 = 0$

$\Rightarrow 6(x^2 - 4) = 0$

$\Rightarrow (x - 2)(x + 2) = 0$

$\Rightarrow x = -2, 2$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

and $f'(x) < 0$ if $-2 < x < 2$

Thus, f(x) increases on $(-\infty, -2) \cup (2, \infty)$

and f(x) is decreasing on interval $x \in (-2,2)$

1 N. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Answer

Given:- Function $f(x) = -2x^3 - 9x^2 - 12x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = \frac{d}{dx}(-2x^3 - 9x^2 - 12x + 1)$

$\Rightarrow f'(x) = -6x^2 - 18x - 12$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow -6x^2 - 18x - 12 = 0$

$\Rightarrow 6x^2 + 18x + 12 = 0$

$\Rightarrow 6(x^2 + 3x + 2) = 0$

$\Rightarrow 6(x^2 + 2x + x + 2) = 0$

$\Rightarrow x^2 + 2x + x + 2 = 0$

$\Rightarrow (x + 2)(x + 1) = 0$

$\Rightarrow x = -1, -2$

clearly, $f'(x) > 0$ if $x < -2$ and $x > -1$

and $f'(x) < 0$ if $-2 < x < -1$

Thus, f(x) increases on $(-\infty, -2) \cup (-1, \infty)$

and f(x) is decreasing on interval $x \in (-2, -1)$

1 0. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = (x - 1)(x - 2)^2$

Answer

Given:- Function $f(x) = (x - 1)(x - 2)^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = (x - 1)(x - 2)^2$

$\Rightarrow f'(x) = \frac{d}{dx}((x-1)(x-2)^2)$

$\Rightarrow f'(x) = (x - 2)^2 + 2(x - 2)(x - 1)$

$\Rightarrow f'(x) = (x - 2)(x - 2 + 2x - 2)$

$\Rightarrow f'(x) = (x - 2)(3x - 4)$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow (x - 2)(3x - 4) = 0$

$\Rightarrow x = 2, \frac{4}{3}$

clearly, $f'(x) > 0$ if $x < \frac{4}{3}$ and $x > 2$

and $f'(x) < 0$ if $\frac{4}{3} < x < 2$

Thus, f(x) increases on $(-\infty, \frac{4}{3}) \cup (2, \infty)$

and f(x) is decreasing on interval $x \in (\frac{4}{3}, 2)$

1 P. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^3 - 12x^2 + 36x + 17$

Answer

Given:- Function $f(x) = x^3 - 12x^2 + 36x + 17$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^3 - 12x^2 + 36x + 17$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 12x^2 + 36x + 17)$

$\Rightarrow f'(x) = 3x^2 - 24x + 36$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 24x + 36 = 0$

$\Rightarrow 3(x^2 - 8x + 12) = 0$

$\Rightarrow 3(x^2 - 6x - 2x + 12) = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow (x - 6)(x - 2) = 0$

$\Rightarrow x = 2, 6$

clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

and $f'(x) < 0$ if $2 < x < 6$

Thus, f(x) increases on $(-\infty, 2) \cup (6, \infty)$

and f(x) is decreasing on interval $x \in (2, 6)$

1 Q. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 2x^3 - 24x + 7$

Answer

Given:- Function $f(x) = 2x^3 - 24x + 7$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 2x^3 - 24x + 7$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 7)$

$\Rightarrow f'(x) = 6x^2 - 24$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 6x^2 - 24 > 0$

$\Rightarrow x^2 < \frac{24}{6}$

$\Rightarrow x^2 < 4$

$\Rightarrow x < -2, +2$

$\Rightarrow x \in (-\infty,-2)$ and $x \in (2,\infty)$

Thus f(x) is increasing on interval $(-\infty, -2) \cup (2, \infty)$

Again, For f(x) to be increasing, we must have

f'(x) < 0

$\Rightarrow 6x^2 - 24 < 0$

$\Rightarrow x^2 > \frac{24}{6}$

$\Rightarrow x^2 < 4$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

Thus f(x) is decreasing on interval $x \in (-1, \infty)$

1 R. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

Answer

Given:- Function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

$$\Rightarrow f'(x) = \frac{d}{dx}\left(\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11\right)$$

$$\Rightarrow f'(x) = 4 \times \frac{3}{10}x^3 - 3 \times \frac{4}{5}x^2 - 6x + \frac{36}{5}$$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$$\Rightarrow \frac{12}{10}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = 0$$

$$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$$

$\Rightarrow x = 1, -2, 3$

Now, lets check values of f(x) between different ranges

Here points x = 1, –2 , 3 divide the number line into disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$

Lets consider interval $(-\infty, -2)$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

Therefore, $f'(x) < 0$ when $-\infty < x < -2$

Thus, f(x) is strictly decreasing on interval $x \in (-\infty, -2)$

consider interval (–2, 1)

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

Therefore, $f'(x) > 0$ when $-2 < x < 1$

Thus, f(x) is strictly increases on interval $x \in (-2, 1)$

Now, consider interval (1, 3)

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

Therefore, $f'(x) < 0$ when $1 < x < 3$

Thus, f(x) is strictly decreases on interval $x \in (1, 3)$

finally, consider interval $(3, \infty)$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

Therefore, $f'(x) > 0$ when $x > 3$

Thus, f(x) is strictly increases on interval $x \in (3, \infty)$

1 S. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^4 - 4x$

Answer

Given:- Function $f(x) = x^4 - 4x$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^4 - 4x$

$\Rightarrow f'(x) = \frac{d}{dx}(x^4 - 4x)$

$\Rightarrow f'(x) = 4x^3 - 4$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 4x^3 - 4 = 0$

$\Rightarrow 4(x^3 - 1) = 0$

$\Rightarrow x = 1$

clearly, $f'(x) > 0$ if $x > 1$

and $f'(x) < 0$ if $x < 1$

Thus, f(x) increases on $(1, \infty)$

and f(x) is decreasing on interval $x \in (-\infty, 1)$

1 T. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

Answer

Given:- Function $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

$$\Rightarrow f'(x) = \frac{d}{dx}\left(\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7\right)$$

$\Rightarrow f'(x) = x^3 + 2x^2 - 5x - 6$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow x^3 + 2x^2 - 5x - 6 = 0$

$\Rightarrow (x+1)(x - 2)(x + 3) = 0$

$\Rightarrow x = -1, 2, -3$

clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

and $f'(x) < 0$ if $x < -3$ and $-3 < x < -1$

Thus, f(x) increases on $(-3, -1) \cup (2, \infty)$

and f(x) is decreasing on interval $(\infty, -3) \cup (-1, 2)$

1 U. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

Answer

Given:- Function $f(x) = x^4 - 4x^3 + 4x^2 + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

$\Rightarrow f'(x) = \frac{d}{dx}(x^4 - 4x^3 + 4x^2 + 15)$

$\Rightarrow f'(x) = 4x^3 - 12x^2 + 8x$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 4x^3 - 12x^2 + 8x = 0$

$\Rightarrow 4(x^3 - 3x^2 + 2x) = 0$

$\Rightarrow x(x^2 - 3x + 2) = 0$

$\Rightarrow x(x^2 - 2x - x + 2) = 0$

$\Rightarrow x(x - 2)(x - 1)$

$\Rightarrow x = 0, 1, 2$

clearly, f'(x) > 0 if 0 < x < 1 and x > 2

and f'(x) < 0 if x < 0 and 1 < x < 2

Thus, f(x) increases on (0, 1) ∪ (2, ∞)

and f(x) is decreasing on interval (–∞, 0) ∪ (1, 2)

1 V. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}},\ x > 0$$

Answer

Given:- Function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$$

$$\Rightarrow f'(x) = \frac{d}{dx}(5x^{\frac{3}{2}} - 3x^{\frac{5}{2}})$$

$$\Rightarrow f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

$$\Rightarrow f'(x) = \frac{15}{2}x^{\frac{1}{2}}(1 - x)$$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$$\Rightarrow \frac{15}{2}x^{\frac{1}{2}}(1 - x) = 0$$

$$\Rightarrow x^{\frac{1}{2}}(1 - x) = 0$$

$\Rightarrow x = 0, 1$

Since $x > 0$, therefore only check the range on the positive side of the number line.

clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, f(x) increases on (0, 1)

and f(x) is decreasing on interval $x \in (1, \infty)$

1 W. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^8 + 6x^2$

Answer

Given:- Function $f(x) = x^8 + 6x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^8 + 6x^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}(8x^7 + 12x)$$

$\Rightarrow f'(x) = 8x^7 + 12x$

$\Rightarrow f'(x) = 4x(2x^6 + 3)$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 4x(2x^6 + 3) = 0$

$\Rightarrow x(2x^6 + 3) = 0$

$\Rightarrow x = 0, \sqrt[\frac{1}{6}]{-\frac{3}{2}}$

Since $x = \sqrt[\frac{1}{6}]{-\frac{3}{2}}$ is a complex number, therefore only check range on 0 sides of number line.

clearly, $f'(x) > 0$ if $x > 0$

and $f'(x) < 0$ if $x < 0$

Thus, f(x) increases on $(0, \infty)$

and f(x) is decreasing on interval $x \in (-\infty, 0)$

1 X. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = x^3 - 6x^2 + 9x + 15$

Answer

Given:- Function $f(x) = x^3 - 6x^2 + 9x + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^3 - 6x^2 + 9x + 15$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 9x + 15)$

$\Rightarrow f'(x) = 3x^2 - 12x + 9$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow 3(x^2 - 4x + 3) = 0$

$\Rightarrow 3(x^2 - 3x - x + 3) = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 1, 3$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

and $f'(x) < 0$ if $1 < x < 3$

Thus, f(x) increases on $(-\infty, 1) \cup (3, \infty)$

and f(x) is decreasing on interval $x \in (1, 3)$

1 Y. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = \{x (x - 2)\}^2$

Answer

Given:- Function $f(x)= \{x (x - 2)\}^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x)= \{x (x - 2)\}^2$

$\Rightarrow f(x)= \{[x^2-2x]\}^2$

$\Rightarrow f'(x) = \frac{d}{dx}([x^2 - 2x]^2)$

$\Rightarrow f'(x)= 2(x^2-2x)(2x-2)$

$\Rightarrow f'(x)= 4x(x-2)(x-1)$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 4x(x-2)(x-1)= 0$

$\Rightarrow x(x-2)(x-1)= 0$

$\Rightarrow x =0, 1, 2$

Now, lets check values of f(x) between different ranges

Here points x = 0, 1, 2 divide the number line into disjoint intervals namely, $(-\infty, 0)$,(0, 1), (1, 2) and $(2, \infty)$

Lets consider interval $(-\infty, 0)$ and (1, 2)

In this case, we have $x(x-2)(x-1)< 0$

Therefore, $f'(x) < 0$ when $x < 0$ and $1< x < 2$

Thus, f(x) is strictly decreasing on interval $(-\infty, 0)\cup(1, 2)$

Now, consider interval (0, 1) and $(2, \infty)$

In this case, we have $x(x-2)(x-1)> 0$

Therefore, $f'(x) > 0$ when $0< x < 1$ and $x < 2$

Thus, f(x) is strictly increases on interval $(0, 1)\cup(2, \infty)$

1 Z. Question

Find the intervals in which the following functions are increasing or decreasing.

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Answer

Given:- Function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$\Rightarrow f'(x) = \frac{d}{dx}(3x^4 - 4x^3 - 12x^2 + 5)$

$\Rightarrow f'(x) = 12x^3 - 12x^2 - 24x$

$\Rightarrow f'(x) = 12x(x^2 - x - 2)$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 12x(x^2 - x - 2)> 0$

$\Rightarrow x(x^2 - 2x + x - 2) > 0$

$\Rightarrow x(x - 2)(x + 1) > 0$

$\Rightarrow -1 < x < 0$ and $x > 2$

$\Rightarrow x \in (-1,0)\cup(2, \infty)$

Thus f(x) is increasing on interval $(-1,0)\cup(2, \infty)$

Again, For f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow 12x(x^2 - x - 2)< 0$

$\Rightarrow x(x^2 - 2x + x - 2) < 0$

$\Rightarrow x(x - 2)(x + 1) < 0$

$\Rightarrow -\infty < x < -1$ and $0 < x < 2$

$\Rightarrow x \in (-\infty, -1) \cup (0, 2)$

Thus f(x) is decreasing on interval $(-\infty, -1) \cup (0, 2)$

1 A1. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

Answer

Given:- Function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

$$\Rightarrow f(x) = \frac{d}{dx}\left(\frac{3}{2}x^4 - 4x^3 - 45x^2 + 51\right)$$

$\Rightarrow f'(x) = 6x^3 - 12x^2 - 90x$

$\Rightarrow f'(x) = 6x(x^2 - 2x - 15)$

$\Rightarrow f'(x) = 6x(x^2 - 5x + 3x - 15)$

$\Rightarrow f'(x) = 6x(x - 5)(x + 3)$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 6x(x - 5)(x + 3) > 0$

$\Rightarrow x(x - 5)(x + 3) > 0$

$\Rightarrow -3 < x < 0$ or $5 < x < \infty$

$\Rightarrow x \in (-3,0) \cup (5, \infty)$

Thus f(x) is increasing on interval $(-3,0) \cup (5, \infty)$

Again, For f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow 6x(x - 5)(x + 3) > 0$

$\Rightarrow x(x - 5)(x + 3) > 0$

$\Rightarrow -\infty < x < -3$ or $0 < x < 5$

$\Rightarrow x \in (-\infty, -3) \cup (0, 5)$

Thus f(x) is decreasing on interval $(-\infty, -3) \cup (0, 5)$

1 B1. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \log(2 + x) - \frac{2x}{2+x}$$

Answer

Given:- Function $f(x) = \log(2 + x) - \frac{2x}{2+x}$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \log(2 + x) - \frac{2x}{2 + x}$$

$$\Rightarrow f'(x) = \frac{d}{dx}\left(\log(2 + x) - \frac{2x}{2+x}\right)$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2-2x \times 1}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$$\Rightarrow \frac{x-2}{(2+x)^2} > 0$$

$\Rightarrow (x - 2) > 0$

$\Rightarrow 2 < x < \infty$

$\Rightarrow x \in (2, \infty)$

Thus f(x) is increasing on interval $(2, \infty)$

Again, For f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow \frac{x-2}{(2+x)^2} < 0$

$\Rightarrow (x - 2) < 0$

$\Rightarrow -\infty < x < 2$

$\Rightarrow x \in (-\infty, 2)$

Thus f(x) is decreasing on interval $(-\infty, 2)$

2. Question

Determine the values of x for which the function $f(x) = x^2 - 6x + 9$ is increasing or decreasing. Also, find the coordinates of the point on the curve $y = x^2 - 6x + 9$ where the normal is parallel to the liney = x + 5.

Answer

Given:- Function $f(x) = x^2 - 6x + 9$ and a line parallel to $y = x + 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^2 - 6x + 9$

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 9)$

$\Rightarrow f'(x) = 2x - 6$

$\Rightarrow f'(x) = 2(x - 3)$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 2(x - 3) = 0$

$\Rightarrow (x - 3) = 0$

$\Rightarrow x = 3$

clearly, $f'(x) > 0$ if $x > 3$

and $f'(x) < 0$ if $x < 3$

Thus, f(x) increases on $(3, \infty)$

and f(x) is decreasing on interval $x \in (-\infty, 3)$

Now, lets find coordinates of point

Equation of curve is

$f(x) = x^2 - 6x + 9$

slope of this curve is given by

$\Rightarrow m_1 = \frac{dy}{dx}$

$\Rightarrow m_1 = \frac{d}{dx}(x^2 - 6x + 9)$

$\Rightarrow m_1 = 2x - 6$

and Equation of line is

$y = x + 5$

slope of this curve is given by

$\Rightarrow m_2 = \frac{dy}{dx}$

$\Rightarrow m_2 = \frac{d}{dx}(x + 5)$

$\Rightarrow m_2 = 1$

Since slope of curve (i.e slope of its normal) is parallel to line

Therefore, they follow the relation

$\Rightarrow \frac{-1}{m_1} = m_2$

$\Rightarrow \frac{-1}{2x-6} = 1$

$\Rightarrow 2x - 6 = -1$

$\Rightarrow x = \frac{5}{2}$

Thus putting the value of x in equation of curve, we get

$\Rightarrow y = x^2 - 6x + 9$

$\Rightarrow y = (\frac{5}{2})^2 - 6\left(\frac{5}{2}\right) + 9$

$\Rightarrow y = \frac{25}{4} - 15 + 9$

$\Rightarrow y = \frac{25}{4} - 6$

$\Rightarrow y = \frac{1}{4}$

Thus the required coordinates is $(\frac{5}{2}, \frac{1}{4})$

3. Question

Find the intervals in which f(x) = sin x – cos x, where 0 < x < 2π is increasing or decreasing.

Answer

Given:- Function f(x) = sin x – cos x, 0 < x < 2π

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = sin x – cos x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x - \cos x)$

⇒ f'(x) = cos x + sin x

For f(x) lets find critical point, we must have

⇒ f'(x) = 0

⇒ cos x + sin x = 0

⇒ tan(x) = –1

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Here these points divide the angle range from 0 to 2∏ since we have x as angle

clearly, f'(x) > 0 if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

and f'(x) < 0 if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, f(x) increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

and f(x) is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

4. Question

Show that f(x) = e^{2x} is increasing on R.

Answer

Given:- Function f(x) = e^{2x}

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = e^{2x}

$\Rightarrow f'(x) = \frac{d}{dx}(e^{2x})$

⇒ f'(x) = $2e^{2x}$

For f(x) to be increasing, we must have

⇒ f'(x) > 0

⇒ $2e^{2x}$ > 0

$\Rightarrow e^{2x} > 0$

since, the value of e lies between 2 and 3

so, whatever be the power of e (i.e x in domain R) will be greater than zero.

Thus f(x) is increasing on interval R

5. Question

Show that $f(x) = e^{\frac{1}{x}}$, x ≠ 0 is a decreasing function for all x ≠ 0.

Answer

Given:- Function $f(x) = e^{\frac{1}{x}}$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = e^{\frac{1}{x}}$

$\Rightarrow f'(x) = \frac{d}{dx}\left(e^{\frac{1}{x}}\right)$

$\Rightarrow f'(x) = e^{\frac{1}{x}}.\left(\frac{-1}{x^2}\right)$

$\Rightarrow f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$

As given x ∈ R , x ≠ 0

$\Rightarrow \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$

Their ratio is also greater than 0

$\Rightarrow \frac{e^{\frac{1}{x}}}{x^2} > 0$

$\Rightarrow -\frac{e^{\frac{1}{x}}}{x^2} < 0$; as by applying –ve sign change in comparision sign

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all x ≠ 0

6. Question

Show that $f(x) = \log_a x$, 0 < a < 1 is a decreasing function for all x > 0.

Answer

Given:- Function $f(x) = \log_a x$, $0 < a < 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a,b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a,b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = \log_a x$, $0 < a < 1$

$\Rightarrow f'(x) = \frac{d}{dx}(\log_a x)$

$\Rightarrow f'(x) = \frac{1}{x \log a}$

As given $0 < a < 1$

$\Rightarrow \log(a) < 0$

and for $x > 0$

$\Rightarrow \frac{1}{x} > 0$

Therefore f'(x) is

$\Rightarrow \frac{1}{x \log a} < 0$

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all $x > 0$

7. Question

Show that f(x) = sin x is increasing on (0, π/2) and decreasing on (π/2, π) and neither increasing nor decreasing in (0, π).

Answer

Given:- Function f(x) = sin x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a,b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a,b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$

⇒ f'(x) = cosx

Taking different region from 0 to 2π

a) let $x \in (0,\frac{\pi}{2})$

⇒ cos(x) > 0

⇒ f'(x) > 0

Thus f(x) is increasing in $(0,\frac{\pi}{2})$

b) let $x \in (\frac{\pi}{2},\pi)$

⇒ cos(x) < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in $(\frac{\pi}{2},\pi)$

Therefore, from above condition we find that

⇒ f(x) is increasing in $(0,\frac{\pi}{2})$ and decreasing in $(\frac{\pi}{2},\pi)$

Hence, condition for f(x) neither increasing nor decreasing in (0,π)

8. Question

Show that f(x) = log sin x is increasing on (0, π/2) and decreasing on (π/2, π).

Answer

Given:- Function f(x) = log sin x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a,b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a,b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = log sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\log \sin x)$

$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$

⇒ f'(x) = cot(x)

Taking different region from 0 to π

a) let $x \in (0,\frac{\pi}{2})$

⇒ cot(x) > 0

⇒ f'(x) > 0

Thus f(x) is increasing in $(0, \frac{\pi}{2})$

b) let $x \in (\frac{\pi}{2}, \pi)$

⇒ cot(x) < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in $(\frac{\pi}{2}, \pi)$

Hence proved

9. Question

Show that f(x) = x – sin x is increasing for all x ϵ R.

Answer

Given:- Function f(x) = x – sin x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = x – sin x

$\Rightarrow f'(x) = \frac{d}{dx}(x - \sin x)$

⇒ f'(x) = 1 – cos x

Now, as given

x ϵ R

⇒ –1 < cosx < 1

⇒ –1 > cos x > 0

⇒ f'(x) > 0

hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval x ∈ R

10. Question

Show that f(x) = x^3 – 15x^2 + 75x – 50 is an increasing function for all x ϵ R.

Answer

Given:- Function f(x) = x^3 – 15x^2 + 75x – 50

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^3 - 15x^2 + 75x - 50$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 15x^2 + 75x - 50)$

$\Rightarrow f'(x) = 3x^2 - 30x + 75$

$\Rightarrow f'(x) = 3(x^2 - 10x + 25)$

$\Rightarrow f'(x) = 3(x - 5)^2$

Now, as given

$x \in R$

$\Rightarrow (x - 5)^2 > 0$

$\Rightarrow 3(x - 5)^2 > 0$

$\Rightarrow f'(x) > 0$

hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

11. Question

Show that $f(x) = \cos^2 x$ is a decreasing function on (0, π/2).

Answer

Given:- Function $f(x) = \cos^2 x$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = \cos^2 x$

$\Rightarrow f'(x) = \frac{d}{dx}(\cos^2 x)$

$\Rightarrow f'(x) = 3\cos x(-\sin x)$

$\Rightarrow f'(x) = -2\sin(x)\cos(x)$

⇒ f′(x) = –sin2x ; as sin2A = 2sinA cosA

Now, as given

$x \in \left(0, \frac{\pi}{2}\right)$

⇒ 2x ∈ (0, π)

⇒ Sin(2x)> 0

⇒ –Sin(2x)< 0

⇒ f′(x) < 0

hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(0, \frac{\pi}{2}\right)$

Hence proved

12. Question

Show that f(x) = sin x is an increasing function on (–π/2, π/2).

Answer

Given:- Function f(x) = sin x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f′(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f′(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f′(x)

(iii) Put f′(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = sin x

⇒ $f'(x) = \frac{d}{dx}(\sin x)$

⇒ f′(x) = cosx

Now, as given

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

That is 4th quadrant, where

⇒ cosx> 0

⇒ f′(x) > 0

hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

13. Question

Show that f(x) = cos x is a decreasing function on (0, π), increasing in (–π, 0) and neither increasing nor decreasing in (–π, π).

Answer

Given:- Function f(x) = cos x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = cos x

⇒ $f'(x) = \frac{d}{dx}(\cos x)$

⇒ f'(x) = –sinx

Taking different region from 0 to 2π

a) let $x \in (0, \pi)$

⇒ sin(x) > 0

⇒ –sinx < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in $(0, \pi)$

b) let $x \in (-\pi, 0)$

⇒ sin(x) < 0

⇒ –sinx > 0

⇒ f'(x) > 0

Thus f(x) is increasing in $(-\pi, 0)$

Therefore, from above condition we find that

⇒ f(x) is decreasing in $(0, \pi)$ and increasing in $(-\pi, 0)$

Hence, condition for f(x) neither increasing nor decreasing in (–π,π)

14. Question

Show that f(x) = tan x is an increasing function on (–π/2, π/2).

Answer

Given:- Function f(x) = tan x

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = tan x

$\Rightarrow f'(x) = \frac{d}{dx}(\tan x)$

$\Rightarrow f'(x) = \sec^2 x$

Now, as given

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

That is 4^{th} quadrant, where

$\Rightarrow \sec^2 x > 0$

$\Rightarrow f'(x) > 0$

hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

15. Question

Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is a decreasing function on the interval (π/4, π /2).

Answer

Given:- Function $f(x) = \tan^{-1}(\sin x + \cos x)$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = \tan^{-1}(\sin x + \cos x)$

$\Rightarrow f'(x) = \frac{d}{dx}(\tan^{-1}(\sin x + \cos x))$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$\Rightarrow\ f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$

Now, as given

$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

⇒ Cosx – sinx< 0 ; as here cosine values are smaller than sine values for same angle

$\Rightarrow \frac{\cos x - \sin x}{2(1+\sin x \cos x)} < 0$

⇒ f'(x) < 0

hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

16. Question

Show that the function $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

Answer

Given:- Function $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$

$\Rightarrow f'(x) = \frac{d}{dx}\left\{\sin\left(2x + \frac{\pi}{4}\right)\right\}$

$\Rightarrow\ f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$

$\Rightarrow\ f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$

Now, as given

$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

$\Rightarrow \frac{3\pi}{8} < x < \frac{5\pi}{8}$

$\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$

$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2};$

as here $2x+\frac{\pi}{4}$ lies in 3rd quadrant

$\Rightarrow \cos\left(2x+\frac{\pi}{4}\right) < 0$

$\Rightarrow 2\cos\left(2x+\frac{\pi}{4}\right) < 0$

⇒ f'(x) < 0

hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

17. Question

Show that the function f(x) = $\cot^{-1}$ (sin x + cos x) is decreasing on (0, π/4) and increasing on (π/4, π/2).

Answer

Given:- Function f(x) = $\cot^{-1}$ (sin x + cos x)

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a,b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a,b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = $\cot^{-1}$ (sin x + cos x)

$\Rightarrow f'(x) = \frac{d}{dx}\{\cot^{-1}(\sin x + \cos x)\}$

$$\Rightarrow f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1+\sin x \cos x)}$$

Now, as given

$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

⇒ Cosx – sinx< 0 ; as here cosine values are smaller than sine values for same angle

$\Rightarrow \frac{\cos x - \sin x}{2(1+\sin x \cos x)} < 0$

⇒ f'(x) < 0

hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

18. Question

Show that f(x) = (x – 1) e^x + 1 is an increasing function for all x > 0.

Answer

Given:- Function f(x) = (x – 1) e^x + 1

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = (x – 1) e^x + 1

$\Rightarrow f'(x) = \frac{d}{dx}((x-1)e^x + 1)$

⇒ f'(x) = e^x + (x – 1) e^x

⇒ f'(x) = e^x(1+ x – 1)

⇒ f'(x) = xe^x

as given

x > 0

⇒ e^x > 0

⇒ xe^x > 0

⇒ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval x > 0

19. Question

Show that the function x^2 – x + 1 is neither increasing nor decreasing on (0, 1).

Answer

Given:- Function f(x) = x^2 – x + 1

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = x^2 - x + 1$

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - x + 1)$

$\Rightarrow f'(x) = 2x - 1$

Taking different region from (0, 1)

a) let $x \in (0, \frac{1}{2})$

$\Rightarrow 2x - 1 < 0$

$\Rightarrow f'(x) < 0$

Thus f(x) is decreasing in $(0, \frac{1}{2})$

b) let $x \in (\frac{1}{2}, 1)$

$\Rightarrow 2x - 1 > 0$

$\Rightarrow f'(x) > 0$

Thus f(x) is increasing in $(\frac{1}{2}, 1)$

Therefore, from above condition we find that

$\Rightarrow$ f(x) is decreasing in $(0, \frac{1}{2})$ and increasing in $(\frac{1}{2}, 1)$

Hence, condition for f(x) neither increasing nor decreasing in (0, 1)

20. Question

Show that $f(x) = x^9 + 4x^7 + 11$ is an increasing function for all $x \in R$.

Answer

Given:- Function $f(x) = x^9 + 4x^7 + 11$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = x^9 + 4x^7 + 11$

$\Rightarrow f'(x) = \frac{d}{dx}(x^9 + 4x^7 + 11)$

$\Rightarrow f'(x) = 9x^8 + 28x^6$

$\Rightarrow f'(x) = x^6(9x^2 + 28)$

as given

$x \in R$

$\Rightarrow x^6 > 0$ and $9x^2 + 28 > 0$

$\Rightarrow x^6(9x^2 + 28) > 0$

$\Rightarrow f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

21. Question

Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on R.

Answer

Given:- Function $f(x) = x^3 - 6x^2 + 12x - 18$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = x^3 - 6x^2 + 12x - 18$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 12x - 18)$

$\Rightarrow f'(x) = 3x^2 - 12x + 12$

$\Rightarrow f'(x) = 3(x^2 - 4x + 4)$

$\Rightarrow f'(x) = 3(x - 2)^2$

as given

$x \in R$

$\Rightarrow (x - 2)^2 > 0$

$\Rightarrow 3(x - 2)^2 > 0$

$\Rightarrow f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

22. Question

State when a function f(x) is said to be increasing on an interval [a, b]. Test whether the function $f(x) = x^2 - 6x + 3$ is increasing on the interval [4, 6].

Answer

Given:- Function $f(x) = f(x) = x^2 - 6x + 3$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find f'(x)

(iii) Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = f(x) = x^2 – 6x + 3

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 3)$

⇒ f'(x) = 2x – 6

⇒ f'(x) = 2(x – 3)

Here A function is said to be increasing on [a,b] if f(x) > 0

as given

x ϵ [4, 6]

⇒ 4 ≤ x ≤ 6

⇒ 1 ≤ (x–3) ≤ 3

⇒ (x – 3) > 0

⇒ 2(x – 3) > 0

⇒ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval x ∈ [4, 6]

23. Question

Show that f(x) = sin x – cos x is an increasing function on (–π /4, π /4)?

Answer

we have,

f(x) = sin x – cos x

f'(x) = cos x + sin x

$= \sqrt{2}(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x)$

$= \sqrt{2}(\frac{\sin \pi}{4}\cos x + \frac{\cos \pi}{4}\sin x)$

$= \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$

Now,

$x \epsilon \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow\ -\frac{\pi}{4} < x < \frac{\pi}{4}$

$\Rightarrow\ 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$

$\Rightarrow \sin 0^\circ < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$

$\Rightarrow 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$

$\Rightarrow \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$

$\Rightarrow f'(x)>0$

Hence, f(x) is an increasing function on (–π /4, π /4)

24. Question

Show that $f(x) = \tan^{-1} x - x$ is a decreasing function on R ?

Answer

we have,

$f(x) = \tan^{-1} x - x$

$f'(x) = \frac{1}{1+x^2} - 1$

$= -\frac{x^2}{1+x^2}$

Now,

$x \in R$

$\Rightarrow x^2 > 0 \text{ and } 1 + x^2 > 0$

$\Rightarrow \frac{x^2}{1+x^2} > 0$

$\Rightarrow -\frac{x^2}{1+x^2} < 0$

$\Rightarrow f'(x) < 0$

Hence, f(x) is an decreasing function for R

25. Question

Determine whether f(x) = x/2 + sin x is increasing or decreasing on (–π /3, π/3) ?

Answer

we have,

$f(x) = -\frac{x}{2} + \sin x$

$= f'(x) = -\frac{1}{2} + \cos x$

Now,

$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

$\Rightarrow\ -\frac{\pi}{3} < x < \frac{\pi}{3}$

$$\Rightarrow \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \cos\left(\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} + \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, f(x) is an increasing function on (–π /3, π /3)

26. Question

Find the interval in which $f(x) = \log(1+x) - \frac{x}{1+x}$ is increasing or decreasing ?

Answer

we have

$$f(x) = \log(1+x) - \frac{x}{1+x}$$

$$f'(x) = \frac{1}{1+x} - \left(\frac{(1+x) - x}{(1+x)^2}\right)$$

$$= \frac{1}{1+x} - \left(\frac{1}{(1+x)^2}\right)$$

$$= \frac{x}{(1+x)^2}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \frac{x}{(1+x)^2} = 0$$

$\Rightarrow x = 0, -1$

Clearly, $f'(x) > 0$ if $x>0$

And $f'(x) < 0$ if $-1 < x < 0$ or $x < -1$

Hence, f(x) increases in $(0,\infty)$, decreases in $(-\infty, -1) \cup (-1, 0)$

27. Question

Find the intervals in which $f(x) = (x + 2)e^{-x}$ is increasing or decreasing ?

Answer

we have,

$f(x) = (x + 2)e^{-x}$

$f'(x) = e^{-x} - e^{-x} (x+2)$

$= e^{-x} (1 - x - 2)$

$= - e^{-x} (x+1)$

Critical points

$f'(x) = 0$

$\Rightarrow - e^{-x} (x + 1) = 0$

$\Rightarrow x = - 1$

Clearly $f'(x) > 0 \text{ if } x < -1$

$f'(x) < 0 \text{ if } x > -1$

Hence f(x) increases in (–∞,–1), decreases in (–1, ∞)

28. Question

Show that the function f given by $f(x) = 10^x$ is increasing for all x ?

Answer

we have,

$f(x) = 10^x$

$\therefore f'(x) = 10^x \log 10$

Now,

$x \in R$

$\Rightarrow 10^x > 0$

$\Rightarrow 10^x \log 10 > 0$

$\Rightarrow f'(x)>0$

Hence, f(x) in an increasing function for all x

29. Question

Prove that the function f given by f(x) = x – [x] is increasing in (0, 1) ?

Answer

we have,

f(x) = x – [x]

$\therefore f'(x) = 1 > 0$

∴ f(x) is an increasing function on (0,1)

30. Question

Prove that the following function is increasing on r?

i. $f(x) = 3x^5 + 40x^3 + 240x$

ii. $f(x) = 4x^3 - 18x^2 + 27x - 27$

Answer

(i) we have

$f(x) = 3x^5 + 40x^3 + 240x$

$\therefore f'(x) = 15x^4 + 120x^2 + 240$

$= 15(x^4 + 8x^2 + 16)$

$= 15(x^2 + 4)^2$

Now,

$x \in R$

$\Rightarrow (x^2+4)^2 > 0$

$\Rightarrow 15(x^2+4)^2 > 0$

$\Rightarrow f'(x) > 0$

Hence, f(x) is an increasing function for all x

(ii) we have

$f(x) = 4x^3 - 18x^2 + 27x - 27$

$\therefore f'(x) = 12x^2 - 36x + 27$

$= 12x^2 - 18x - 18x + 27$

$= 3(2x-3)^2$

Now,

$x \in R$

$\Rightarrow (2x-3)^2 > 0$

$\Rightarrow 3(2x-3)^2 > 0$

$\Rightarrow f'(x) > 0$

Hence, f(x) is an increasing fuction for all x

31. Question

Prove that the function f given by f(x) = log cos x is strictly increasing on (–π/2, 0) and strictly decreasing on (0, π/2) ?

Answer

we have,

$f(x) = \log \cos x$

$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$

In Interval $(0,\frac{\pi}{2})$, tan x > 0 $\Rightarrow -\tan x < 0$

$\therefore f'(x) < 0 \text{ on } \left(0,\frac{\pi}{2}\right)$

$\therefore \text{f is strickly decreasing on } \left(0,\frac{\pi}{2}\right)$

In interval $(\frac{\pi}{2},\pi)$, tan x < 0 $\Rightarrow -\tan x > 0$

$\therefore f'(x) > 0 \text{ on } \left(\frac{\pi}{2},\pi\right)$

32. Question

Prove that the function f given by f(x) = $x^3 - 3x^2 + 4x$ is strictly increasing on R ?

Answer

given $f(x) = x^3 - 3x^2 + 4x$

$\therefore$ f(x) $= 3x^2 - 6x + 4$

$= 3(x^2 - 2x + 1) + 1$

$= 3(x-1)^2 + 1 > 0$ for all $x \in R$

Hence f(x) is strickly increasing on R

33. Question

33 Prove that the function f(x) = cos x is :

i. strictly decreasing on (0, π)

ii. strictly increasing in (π, 2π)

iii. neither increasing nor decreasing in (0, 2 π)

Answer

Given f(x) =cos x

$\therefore f'(x) = -\sin x$

(i) Since for each x $\in(0, \pi)$,sin x > 0

$\Rightarrow \therefore f'(x) < 0$

So f is strictly decreasing in (0, π)

(ii) Since for each x $\in(\pi, 2\pi)$,sin x <0

$\Rightarrow \therefore f'(x) > 0$

So f is strictly increasing in (π,2π)

(iii) Clearly from (1) and (2) above, f is neither increasing nor decreasing in (0,2π)

34. Question

Show that f(x) = x^2 – x sin x is an increasing function on (0, π/2) ?

Answer

We have,

f(x) = x^2– x sinx

f ’(x) = 2x – sin x – x cos x

Now,

x $\in(0, \frac{\pi}{2})$

⇒ 0 $\leq$ sin x $\leq$1, 0 $\leq$ cos x $\leq$1,

⇒ 2x–sin x –x cos x > 0

⇒ f ’(x) ≥ 0

Hence,f(x) is an increasing function on $(0, \frac{\pi}{2})$.

35. Question

Find the value(s) of a for which f(x) = x^3 – ax is an increasing function on R ?

Answer

We have,

f(x) = x^3– ax

$f'(x) = 3x^2 - a$

Given that f(x) is on increasing function

$\therefore f'(x) 0$ for all x $\in$ R

$\Rightarrow 3x^2 - a > 0$ for all x $\in$ R

$\Rightarrow$ a < $3x^2$ for all x $\in$ R

But the last value of $3x^2 = 0$ for x = 0

$\therefore a \le 0$

36. Question

Find the values of b for which the function f(x) = sin x – bx + c is a decreasing function on R ?

Answer

We have,

f(x) = sin x – bx +c

$f'(x) = \cos x - b$

Given that f(x) is on decreasing function on R

$\therefore f'(x) < 0$ for all x $\in$ R

$\Rightarrow \cos x \ - b > 0$ for all x $\in$ R

$\Rightarrow$ b < $\cos x$ for all x $\in$ R

But the last value of cos x in 1

$\therefore b \ge 1$

37. Question

Show that f(x) = x + cos x – a is an increasing function on R for all values of a ?

Answer

We have,

f(x) = x + cos x – a

$$f'(x) = 1 - \sin x = \frac{2\cos^2 x}{2}$$

Now,

x $\in$ R

$\Rightarrow \frac{\cos^2 x}{2} > 0$

$\Rightarrow \frac{2\cos^2 x}{2} > 0$

$\Rightarrow f'(x) > 0$

Hence,f(x) is an increasing function for x $\in$ R

38. Question

Let F defined on [0, 1] be twice differentiable such that | f”(x) ≤ 1 for all x$\in$ [0, 1]. If f(0) = f(1), then show that |f’(x) | < 1 for all x $\in$ [0, 1] ?

Answer

As f(0) = f(1) and f is differentiable, hence by Rolles theorem:

$f'(c) = 0$ for some c $\in$[0,1]

let us now apply LMVT (as function is twice differentiable) for point c and x $\in$[0,1],

hence,

$\frac{|f'(x)-f(c)|}{x-c}$ =f ”(d)

$\Rightarrow \frac{|f'(x)-0|}{x-c}$ =f ”(d)

$\Rightarrow \frac{|f'(x)|}{x-c}$ =f ”(d)

A given that | f ”(d)| <=1 for x $\in$[0,1]

$\Rightarrow \frac{|f'(x)|}{x-c} \leq 1$

$\Rightarrow |f'(x)| \leq x - c$

Now both x and c lie in [0,1], hence $x - c$ $\in$[0,1]

39. Question

Find the intervals in which f(x) is increasing or decreasing :

i. f(x) = x |x|, x ϵ R

ii. f(x) = sin x + |sin x|, 0 < x ≤ 2 π

iii. f(x) = sin x (1 + cos x), 0 < x < π/2

Answer

(i): Consider the given function,

f(x) = x |x|, x ϵ R

$\Rightarrow f(x) = \{^{-x^2, x<0}_{x^2, x>0}$

$\Rightarrow f'(x) = \{^{-2x, x<0}_{2x, x>0}$

⇒ f’(x)> 0

Therefore, f(x) is an increasing function for all real values.

(ii): Consider the given function,

f(x) = sin x +|sin x|, 0< x$\leq$ 2π

$\Rightarrow f(x) = \{^{2\sin x, 0 < x \leq \pi}_{0, \pi < x \leq 2\pi}$

$\Rightarrow f'(x) = \{^{2\cos x, 0 < x \leq \pi}_{0, \pi < x \leq 2\pi}$

The function 2cos x will be positive between (0,$\frac{\pi}{2}$)

Hence the function f(x) is increasing in the interval (0,$\frac{\pi}{2}$)

The function 2cos x will be negative between ($\frac{\pi}{2}$, π)

Hence the function f(x) is decreasing in the interval ($\frac{\pi}{2}$, π)

The value of f’(x)= 0, when, $\pi < x \leq 2\pi$

Therefore, the function f(x) is neither increasing nor decreasing in the interval (π, 2π)

(iii): consider the function,

$f(x) = \sin x(1 + \cos x),\ 0 < x < \frac{\pi}{2}$

$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$

$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$

$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$

$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

for f(x) to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

$\Rightarrow 0 < x < \frac{\pi}{3}$

So, f(x) to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$

$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

So, f(x) is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

MCQ

1. Question

Mark the correct alternative in the following:

The interval of increase of the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ is

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(1, \infty)$

D. $(-\infty, 1)$

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$$f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$$

$$d\left(\frac{f(x)}{dx}\right) = 1 - e^x = f'(x)$$

Now

$f'(x) > 0$

$\Rightarrow 1-e$

x>0

X<0

$x \in (-\infty, 0)$

2. Question

Mark the correct alternative in the following:

The function f(x) = $\cos^{-1}$ x + x increases in the interval.

A. (1, ∞)

B. (–1, ∞)

C. (–∞, ∞)

D. (0, ∞)

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

Given:-

f(x) = $\cos^{-1}$ x + x

$$d\left(\frac{f(x)}{dx}\right) = \frac{x^2}{1+x^2} = f'(x)$$

Now

f'(x)>0

$$\Rightarrow \frac{x^2}{1+x^2} > 0$$

x∈R

⇒x∈(–∞, ∞)

3. Question

Mark the correct alternative in the following:

The function f(x) = x^x decreases on the interval.

A. (0, e)

B. (0, 1)

C. (0, 1/e)

D. (1/e, e)

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

Given:-

f(x) = x^x

$$d\left(\frac{f(x)}{dx}\right) = x^x(1 + \log x) = f'(x)$$

now for decreasing

$f'(x)<0$

$\Rightarrow x^x(1+\log x)<0$

$\Rightarrow (1+\log x)<0$

$\Rightarrow \log x<-1$

$\Rightarrow x<e^{-1}$

$x \in \left(0, \frac{1}{e}\right)$

4. Question

Mark the correct alternative in the following:

The function $f(x) = 2\log(x - 2) - x^2 + 4x + 1$ increases on the interval.

A. (1, 2)

B. (2, 3)

C. ((1, 3)

D. (2, 4)

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$f(x) = 2\log(x - 2) - x^2 + 4x + 1$

$d\left(\frac{f(x)}{dx}\right) = \frac{2}{x-2} - 2x + 4 = f'(x)$

$\Rightarrow f'(x) = -\frac{2(x-1)(x-3)}{x-2}$

now for increasing

$f'(x)>0$

$\Rightarrow -\frac{2(x-1)(x-3)}{x-2} < 0$

$x - 3<0$ and $x-2>0$

$x < 3$ and $x>2$

$x \in (2,3)$

5. Question

Mark the correct alternative in the following:

If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1, 2], then k lies in the interval.

A. $(-\infty, 4)$

B. $(4, \infty)$

C. $(-\infty, 8)$

D. (8, ∞)

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

$f(x) = 2x^2 - kx + 5$

$$d\left(\frac{f(x)}{dx}\right) = 4x - k = f'(x)$$

$f'(x)>0$

$\Rightarrow 4x-k>0$

$\Rightarrow K<4x$

For x=1

$\Rightarrow K<4$

6. Question

Mark the correct alternative in the following:

Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the set R. Then, a and b satisfy.

A. $a^2 - 3b - 15 > 0$

B. $a^2 - 3b + 15 > 0$

C. $a^2 - 3b + 15 < 0$

D. a > 0 and b > 0

Answer

Formula:- (i) $ax^2+bx+c>0$ for all x $\Rightarrow a>0$ and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all x $\Rightarrow a<0$ and $b^2-4ac<0$

(iii)The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

Given:-

$f(x) = x^3 + ax^2 + bx + 5\sin^2 x$

$$d\left(\frac{f(x)}{dx}\right) = 3x^2 + 2ax + b + 5\sin^2 x = f'(x)$$

For increasing function f'(x)>0

$3x^2+2ax+b+5\sin 2x>0$

Then

$3x^2+2ax+b-5<0$

And $b^2-4ac<0$

$\Rightarrow 4a^2-12(b-5)<0$

$\Rightarrow a^2-3b+15<0$

$\Rightarrow a^2 - 3b + 15 < 0$

7. Question

Mark the correct alternative in the following:

The function $f(x)=\log_e\left(x^3+\sqrt{x^6+1}\right)$ is of the following types:

A. even and increasing

B. odd and increasing

C. even and decreasing

D. odd and decreasing

Answer

Formula:- (i)if f(-x)=f(x) then function is even

(ii) if f(-x)=-f(x) then function is odd

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

Given:-

f(x)=$\log_e(x^3+\sqrt{x^6+1})$

$$d\left(\frac{f(x)}{dx}\right)=\frac{1}{x^3(x^6+1)^{\frac{1}{2}}}\left(3x^2+\frac{6x^5}{2(x^6+1)^{\frac{1}{2}}}\right)$$

$f'(x)>0$

hence function is increasing function

f(-x)=-log($\log_e(x^3+\sqrt{x^6+1}$)

⇒f(-x)=-f(x) is odd function

8. Question

Mark the correct alternative in the following:

If the function f(x) = 2tanx + (2a + 1) $\log_e$ |sec x| + (a – 2) x is increasing on R, then

A. $a\in\left(\frac{1}{2},\infty\right)$

B. $a\in\left(-\frac{1}{2},\frac{1}{2}\right)$

C. $a=\frac{1}{2}$

D. $a\in R$

Answer

Formula:- (i) $ax^2+bx+c>0$ for all x ⇒a>0 and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all x ⇒a<0 and $b^2-4ac<0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

Given:-

f(x) = 2tanx+(2a+1)$\log_e$ |sec x|+(a – 2)x

$$d\left(\frac{f(x)}{dx}\right) = 2\sec^2 x + \frac{(2a+1)\sec x.\tan x}{\sec x} + (a-2) = f'(x)$$

$\Rightarrow f'(x)=2\sec^2 x+ (2a+1)\tan x + (a-2)$

$\Rightarrow f'(x)=2(\tan^2+1) + (2a+1).\tan x +(a-2)$

$\Rightarrow f'(x)=2\tan^2 x+2a\tan x+\tan x+a$

For increasing function

f’(x)>0

$\Rightarrow 2\tan^2 x+2a\tan x+\tan x+ a>0$

From formula (i)

$(2a+1)^2-8a<0$

$$\Rightarrow 4\left(a-\frac{1}{2}\right)^2 < 0$$

$$\Rightarrow a = \frac{1}{2}$$

9. Question

Mark the correct alternative in the following:

Let f(x) = $\tan^{-1}$ (g(x)), where g(x) is monotonically increasing for $0 < x < \frac{\pi}{2}$. Then, f(x) is

A. increasing on $\left(0, \frac{\pi}{2}\right)$

B. decreasing on $\left(0, \frac{\pi}{2}\right)$

C. increasing on $\left(0, \frac{\pi}{4}\right)$ and decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

D. none of these

Answer

Formula:-

(i)The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f’(x)>0 for all x$\in$(a,b)

Given:- f(x) = $\tan^{-1}$ (g(x))

$$\frac{d(f(x))}{dx} = \frac{g'(x)}{1+(g(x))^2} = f'(x)$$

For increasing function

f’(x)>0

$x \in \left(0, \frac{\pi}{2}\right)$

10. Question

Mark the correct alternative in the following:

Let f(x) = x^3 – $6x^2$ + 15x + 3. Then,

A. f(x) > 0 for all x ϵR

B. f(x) >f(x + 1) for all x ϵR

C. f(x) in invertible

D. f(x) < 0 for all x ϵ R

Answer

Formula:- (i)The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that f'(x)>0 for all x∈(a,b)

(ii)If f(x) is strictly increasing function on interval [a, b], then f^{-1} exist and it is also a strictly increasing function

Given:- f(x) = x^3 – $6x^2$ + 15x + 3

$\frac{d(f(x))}{dx}$=$3x^2$-12x+15=f'(x)

$\Rightarrow f'(x) = 3(x-2)^2 + \frac{1}{3}$

$\Rightarrow f'(x) = 3(x-2)^2 + \frac{1}{3}$

Therefore f'(x) will increasing

Also $f^{-1}(x)$ is possible

Therefore f(x) is invertible function.

11. Question

Mark the correct alternative in the following:

The function f(x) = $x^2 e^{-x}$ is monotonic increasing when

A. x ϵR – [0, 2]

B. 0 < x < 2

C. 2 < x < ∞

D. x < 0

Answer

f(x) = $x^2 e^{-x}$

$\frac{d(f(x))}{dx}$=xe^{-x}(2-x)=f'(x)

for

f'(x)=0

⇒ $x^2 e^{-x}$=0

⇒x(2-x)=0

x=2,x=0

f(x) is increasing in (0,2)

12. Question

Mark the correct alternative in the following:

Function f(x) = cosx – 2λ x is monotonic decreasing when

A. $\lambda > \frac{1}{2}$

B. $\lambda < \frac{1}{2}$

C. λ < 2

D. λ > 2

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a,b) is that f’(x)<0 for all x∈(a,b)

Given:-

f(x) = cosx – 2λ x

$\frac{d(f(x))}{dx}$=-sinx-2λ =f’(x)

for decreasing function f’(x)<0

-sinx-2λ <0

⇒Sinx+2λ >0

⇒2λ>-sinx

⇒2λ>1

$\Rightarrow \lambda > \frac{1}{2}$

13. Question

Mark the correct alternative in the following:

In the interval (1, 2), function f(x) = 2 |x – 1|+3|x – 2| is

A. monotonically increasing

B. monotonically decreasing

C. not monotonic

D. constant

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreading on (a,b) is that f’(x)<0 for all x∈(a,b)

Given:-

f(x)=2(x-1)+3(2-x)

f(x)=-x+4

$$\frac{d(f(x))}{dx} = -1 = f'(x)$$

Therefore $f'(x) < 0$

Hence decreasing function

14. Question

Mark the correct alternative in the following:

Function $f(x) = x^3 - 27x + 5$ is monotonically increasing when

A. $x < -3$

B. $|x| > 3$

C. $x \le -3$

D. $|x| \ge 3$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$f(x)= x^3 - 27x +5$

$\frac{d(f(x))}{dx} = 3x^2 - 27 = f'(x)$

for increasing function $f'(x)>0$

$3x^2 - 27>0$

$\Rightarrow (x+3)(x-3)>0$

$\Rightarrow |x|>3$

15. Question

Mark the correct alternative in the following:

Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when

A. $x < 2$

B. $x > 2$

C. $x > 3$

D. $1 < x < 2$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a, b) is that $f'(x)<0$ for all $x \in (a,b)$

Given:-

$f(x) = 2x^3 - 9x^2 + 12x + 29$

$\frac{d(f(x))}{dx} = f'(x) = 6(x-1)(x-2)$

for decreasing function $f'(x)<0$

$f'(x)<0$

$\Rightarrow 6(x-1)(x-2)<0$

$\Rightarrow 1<x<2$

16. Question

Mark the correct alternative in the following:

If the function $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then

A. $k < 3$

B. $k \leq 3$

C. $k > 3$

D. $k < 3$

Answer

Formula:- (i) $ax^2+bx+c>0$ for all $x \Rightarrow a>0$ and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all $x \Rightarrow a<0$ and $b^2-4ac<0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$f(x) = kx^3 - 9x^2 + 9x + 3$

$\frac{d(f(x))}{dx} = f'(x) = 3kx^2 - 18x + 9$

for increasing function $f'(x)>0$

$f'(x)>0$

$\Rightarrow 3kx^2-18x+9>0$

$\Rightarrow kx^2-6x+3>0$

using formula (i)

$36-12k<0$

$\Rightarrow k>3$

17. Question

Mark the correct alternative in the following:

$f(x) = 2x - \tan^{-1} x - \log\left\{x + \sqrt{x^2+1}\right\}$ is monotonically increasing when

A. $x > 0$

B. $x < 0$

C. $x \in R$

D. $x \in R - \{0\}$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$f(x) = 2x - \tan^{-1}x - \log\{x + \sqrt{x^2+1}\}$

$$\frac{df(x)}{dx} = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{x^2+1}} = f'(x)$$

For increasing function f'(x)>0

$$\Rightarrow 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{x^2+1}} > 0$$

$x \in R$

18. Question

Mark the correct alternative in the following:

Function f(x) = |x| – |x – 1| is monotonically increasing when

A. x < 0

B. x > 1

C. x < 1

D. 0<x<1

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all $x\in(a,b)$

Given:-

For x<0

f(x)=-1

for 0<x<1

f(x)=2x-1

for x>1

f(x)=1

Hence f(x) will increasing in 0<x<1

19. Question

Mark the correct alternative in the following:

Every invertible function is

A. monotonic function

B. constant function

C. identity function

D. not necessarily monotonic function

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all $x\in(a,b)$

If f(x) is strictly increasing function on interval [a, b], then f^{-1} exist and it is also a strictly increasing function

20. Question

Mark the correct alternative in the following:

In the interval (1, 2), function f(x) = 2|x – 1|+3 |x – 2| is

A. increasing

B. decreasing

C. constant

D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a, b) is that f'(x)<0 for all x∈(a,b)

Given:-

f(x)=2(x-1)+3(2-x)

⇒f(x)=-x+4

$\frac{d(f(x))}{dx}$=f'(x)=-1

Therefore f'(x) <0

Hence decreasing function

21. Question

Mark the correct alternative in the following:

If the function f(x) = cos|x| – 2ax + b increases along the entire number scale, then

A. a = b

B. $a = \frac{1}{2}b$

C. $a \leq -\frac{1}{2}$

D. $a > -\frac{3}{2}$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

Given:-

f(x) = cos|x| – 2ax + b

$$\frac{d(f(x))}{dx} = -\sin x - 2a = f'(x)$$

For increasing f'(x)>0

⇒-sinx-2a>0

⇒2a<-sinx

$\Rightarrow 2a \leq -1$

$\Rightarrow a \leq -\frac{1}{2}$

22. Question

Mark the correct alternative in the following:

The function $f(x)=\frac{x}{1+|x|}$ is

A. strictly increasing

B. strictly decreasing

C. neither increasing nor decreasing

D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

$$f(x)=\frac{x}{1+|x|}$$

For x>0

$$\frac{d(f(x))}{dx}=\frac{1}{1+x^2}=f'(x)$$

For x<0

$$\frac{d(f(x))}{dx}=\frac{1}{1-x^2}=f'(x)$$

Both are increasing for f'(x)>0

23. Question

Mark the correct alternative in the following:

The function $f(x)=\frac{\lambda\sin x+2\cos x}{\sin x+\cos x}$ is increasing, if

A. λ < 1

B. λ > 1

C. λ < 2

D. λ > 2

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

Given:-

$$f(x)=\frac{\lambda\text{sinx}+2\text{cosx}}{\text{sinx}+\text{cosx}}$$

For increasing function f'(x)<0

$$\frac{d(f(x))}{dx}=f'(x)=\frac{\lambda-2}{(\text{sinx}+\text{cosx})^2}>0$$

⇒ λ>2

24. Question

Mark the correct alternative in the following:

Function $f(x) = a^x$ is increasing or R, if

A. $a > 0$

B. $a < 0$

C. $a > 1$

D. $a > 0$

Answer

Let $x_1<x_2$ and both are real number

$a^{x_1} < a^{x_2}$

$\Rightarrow f(x_1)<f(x_2)$

$\Rightarrow x_1<x_2 \in$

only possible on a>1

25. Question

Mark the correct alternative in the following:

Function $f(x) = \log_a x$ is increasing on R, if

A. $0 < a < 1$

B. $a > 1$

C. $a < 1$

D. $a > 0$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

$f(x) = \log_a x$

$$\frac{d(f(x))}{dx} = \frac{1}{x\log_e a} = f'(x)$$

For increasing f'(x)>0

$$\Rightarrow \frac{1}{x\log_e a} > 0$$

For log a>1

26. Question

Mark the correct alternative in the following:

Let $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$ for all $x \in [0, a]$. The, $\phi(x)$

A. increases on [0, a]

B. decreases on [0, a]

C. increases on [–a, 0]

D. decreases on [a, 2a]

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

$\phi(x) = f(x) + f(2a - x)$

$\Rightarrow \phi'(x) = f'(x) - f'(2a - x)$

$\Rightarrow \phi''(x) = f''(x) + f''(2a - x)$

checking the condition

$\phi(x)$ is decreasing in [0,a]

27. Question

Mark the correct alternative in the following:

If the function $f(x) = x^2 - kx + 5$ is increasing on [2, 4], then

A. k ∈ (2, ∞)

B. k∈ (–∞, 2)

C. k ∈ (4, ∞)

D. k∈ (–∞, 4)

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

Given:-

$f(x) = x^2 - kx + 5$

$$\frac{d(f(x))}{dx} = 2x - k = f'(x)$$

For increasing function f'(x)>o

2x-k>0

⇒K<2x

Putting x=2

K<4

⇒ k∈ (–∞, 4)

28. Question

Mark the correct alternative in the following:

The function $f(x) = -\frac{x}{2} + \sin x$ defined on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ is

A. increasing

B. decreasing

C. constant

D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that f'(x)>0 for all x∈(a,b)

Given:-

$f(x) = -\frac{x}{2} + \sin x$

$\frac{d(f(x))}{dx} = -\frac{1}{2} + \cos x = f'(x)$

checking the value of x

$\cos - \frac{1}{2} > 0$

hence increasing

29. Question

Mark the correct alternative in the following:

If the function $f(x) = x^3 - 9k\,x^2 + 27x + 30$ is increasing on R, then

A. $-1 \le k < 1$

B. $k < -1$ or $k > 1$

C. $0 < k < 1$

D. $-1 < k < 0$

Answer

Formula:- (i) $ax^2+bx+c>0$ for all x $\Rightarrow a>0$ and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all x $\Rightarrow a<0$ and $b^2-4ac<0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$f(x) = x^3 - 9k\,x^2 + 27x + 30$

$\frac{d(f(x))}{dx} = f'(x) = 3x^2 - 18kx + 27$

for increasing function $f'(x)>0$

$3x^2-18kx+27>0$

$\Rightarrow x^2-6kx+9>0$

Using formula (i)

$36k^2-36>0$

$\Rightarrow K^2>1$

Therefore $-1 < k < 1$

30. Question

Mark the correct alternative in the following:

The function $f(x) = x^9 + 3x^7 + 64$ is increasing on

A. R

B. $(-\infty, 0)$

C. $(0, \infty)$

D. R_0

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x)>0$ for all $x\in(a,b)$

Given:-

$f(x) = x^9 + 3x^7 + 64$

$$\frac{d(f(x))}{dx} = 9x^8 + 21x^6 = f'(x)$$

For increasing $f'(x)>o$

$\Rightarrow 9x^8+21x^6>0$

$\Rightarrow x\in R$